

Indicator Functions

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Indicator functions

An **indicator function** is a mathematical function that signals whether an element belongs to a specified set, or whether a given logical condition is satisfied. In statistics and epidemiology, indicator functions are ubiquitous. They represent:

- binary variables, which take only the values 0 and 1 (Definition [0.3](#));
- event and censoring indicators in survival analysis, the study of the time until an event such as death, where an observation is censored when follow-up ends before its event is seen;
- membership in subpopulations;
- domain restrictions in integrals and sums.

Despite their conceptual simplicity, notation for indicator functions varies substantially across textbooks, research papers, and subfields. This section summarizes the principal notational conventions.

Definition 0.1 (Indicator function, set indicator). For any subset $A \subseteq \Omega$ of a universal set Ω , the **indicator function** of A , $[1]$ or **set indicator**, is the function $\mathbb{1}_A : \Omega \rightarrow \{0, 1\}$ defined by:

$$\mathbb{1}_A(x) \stackrel{\text{def}}{=} \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Definition 0.2 (Predicate indicator). For any logical proposition or predicate P , the **indicator** of P , or **predicate indicator**, takes the value 1 when P is true and 0 when P is false:

$$\mathbb{1}(P) \stackrel{\text{def}}{=} \begin{cases} 1, & \text{if } P \text{ is true} \\ 0, & \text{if } P \text{ is false} \end{cases}$$

Example 0.1 (Evaluating set and predicate indicators). Consider the real line $\Omega = \mathbb{R}$, the set of nonnegative numbers $A = [0, \infty)$, and a continuous **random variable** Y (defined in the Morrison Lab’s probability notes).

1. Set indicator $\mathbb{1}_A(x)$:

- For $x = 3.5$: since $3.5 \in [0, \infty)$, $\mathbb{1}_A(3.5) = 1$.
- For $x = -2.1$: since $-2.1 \notin [0, \infty)$, $\mathbb{1}_A(-2.1) = 0$.

2. Predicate indicator $\mathbb{1}(Y > 5)$:

- If an observation yields $Y = 7.2$, the predicate $7.2 > 5$ is true, so $\mathbb{1}(7.2 > 5) = 1$.
- If an observation yields $Y = 4.1$, the predicate $4.1 > 5$ is false, so $\mathbb{1}(4.1 > 5) = 0$.

Definition 0.3 (Binary variable). A **binary variable** is a variable that takes only the values 0 and 1.

Example 0.2 (A binary variable is an indicator). In a study of n people, let $s_i = 1$ if person i smokes and $s_i = 0$ otherwise, for $i \in \{1, \dots, n\}$. Then s_i is a binary variable (Definition 0.3). It is also the predicate indicator (Definition 0.1) of the proposition “person i smokes”: $s_i = \mathbb{1}(\text{person } i \text{ smokes})$.

In the same way, every indicator is a binary variable, and every binary variable X is the indicator of its own value being 1: $X = \mathbb{1}(X = 1)$, since both sides are 1 when $X = 1$ and 0 when $X = 0$.

Two primary notational paradigms: set vs. predicate notation

The vast majority of indicator notations belong to one of two families: set notation or predicate notation.

Definition 0.4 (Set notation for indicators). An indicator (Definition 0.1) is written in **set notation** when the set A appears as a subscript, as in $\mathbb{1}_A(x)$.

Definition 0.5 (Predicate notation for indicators). An indicator (Definition 0.1) is written in **predicate notation** when a proposition or predicate P (definition) appears as

its argument, as in $\mathbb{1}(P)$ or $\mathbb{1}(x \in A)$.

Example 0.3 (One indicator in both notations). Let $A = [0, \infty)$, the nonnegative real numbers. The indicator of A is $\mathbb{1}_A(x)$ in set notation, and $\mathbb{1}(x \in A)$ or $\mathbb{1}(x \geq 0)$ in predicate notation (Definition 0.4, Definition 0.5). At $x = 3.5$, all three equal 1; at $x = -2.1$, all three equal 0.

Set notation

In set notation, the indicator is tied to a set A , which appears as a subscript:

- $\mathbf{1}_A(x)$ (bold numeral one)
- $\mathbb{1}_A(x)$ (blackboard bold numeral one)
- $I_A(x)$ (capital letter I)
- $\mathbb{I}_A(x)$ (blackboard bold letter I)
- $\chi_A(x)$ (Greek letter chi, historically termed the *characteristic function*)

When the function is viewed as a mathematical object in its own right (for instance, as an element of an L^p function space, the set of functions f for which $\int |f(x)|^p dx$ is finite), authors often omit the argument x , writing simply $\mathbf{1}_A$, $\mathbb{1}_A$, or I_A .

Predicate notation

In predicate notation, the indicator takes a logical condition, relation, or proposition P directly as its argument or subscript:

- $\mathbb{I}(x \in A)$ or $\mathbb{I}(P)$
- $\mathbf{1}(x \in A)$ or $\mathbf{1}(P)$
- $\mathbb{1}(x \in A)$ or $\mathbb{1}(P)$
- $\mathbb{1}\{x \in A\}$ or $\mathbb{1}\{P\}$
- $I(x \in A)$ or $I(P)$

Predicate notation is especially common in applied statistics and survival analysis, where indicators frequently depend on inequalities (definition) involving random variables, such as $\mathbb{1}(T_i \leq t)$ (an event occurring before time t) or $\mathbb{1}(Y_i = 1)$ (a binary outcome).

Equivalence between paradigms

The two paradigms are connected by evaluating the predicate indicator at the membership statement $x \in A$:

$$\mathbf{1}_A(x) = \mathbb{I}(x \in A)$$

Definition 0.6 (Support of a distribution). The **support** of a discrete distribution [2] is the set of values it gives positive probability. The support of a continuous distribution is the set of values where its density is positive.

Example 0.4 (Supports of two count distributions).

- A fair six-sided die gives each of $1, 2, \dots, 6$ probability $\frac{1}{6} > 0$ and every other value probability 0, so its support is $\{1, 2, 3, 4, 5, 6\}$.
- A count that can be 0, 1, 2, and so on, each with positive probability, such as the number of hospital visits in a year, has support $\mathbb{N}_0 = \{0, 1, 2, \dots\}$.

Set notation is more natural when the underlying set A has a standard name (such as the support of a distribution (Definition 0.6), or a geometric region). Predicate notation is more natural when the condition involves compound inequalities, such as $\mathbb{1}(0 \leq t \leq u)$.

Iverson bracket notation

In 1962, Kenneth Iverson introduced a compact notation in the programming language APL, later popularized in mathematics and computer science by Donald Knuth: the Iverson bracket [3].

Definition 0.7 (Iverson bracket). For any logical proposition P , the **Iverson bracket** of P is [3]

$$[P] \stackrel{\text{def}}{=} \begin{cases} 1, & \text{if } P \text{ is true} \\ 0, & \text{if } P \text{ is false} \end{cases}$$

Example 0.5 (Evaluating Iverson brackets).

- $[3 > 2] = 1$, because $3 > 2$ is true.
- $[2 > 3] = 0$, because $2 > 3$ is false.
- $[4 \in \{1, 2\}] = 0$, because 4 is not an element of $\{1, 2\}$.

i Remark (The Iverson bracket is the predicate indicator)

Remark 0.1 (The Iverson bracket is the predicate indicator). The Iverson bracket $[P]$ is the predicate indicator $\mathbb{1}(P)$ (Definition 0.1) in different notation. Under this notation, set membership is written $[x \in A]$, so $[x \in A] = \mathbb{1}_A(x)$. For example, the values in Example 0.1 become

$$\begin{aligned}[7.2 > 5] &= \mathbb{1}(7.2 > 5) \\ &= 1\end{aligned}$$

and, with $A = [0, \infty)$,

$$\begin{aligned}[-2.1 \in A] &= \mathbb{1}_A(-2.1) \\ &= 0.\end{aligned}$$

Definition 0.8 (Kronecker delta). For integers i and j , the **Kronecker delta** is [4]

$$\delta_{ij} \stackrel{\text{def}}{=} [i = j]$$

that is, $\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$ (Definition 0.7).

Example 0.6 (Evaluating the Kronecker delta).

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$$\begin{aligned}\delta_{11} &= [1 = 1] \\ &= 1.\end{aligned}$$

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$$\begin{aligned}\delta_{12} &= [1 = 2] \\ &= 0.\end{aligned}$$

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$$\begin{aligned}\delta_{22} &= [2 = 2] \\ &= 1.\end{aligned}$$

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$$\begin{aligned}\delta_{23} &= [2 = 3] \\ &= 0.\end{aligned}$$

Strengths and limitations of the Iverson bracket

The primary advantage of the Iverson bracket is algebraic conciseness: it turns a sum or integral over a subset into a sum or integral over a larger, fixed set. For example, if U is a [finite set](#) that contains A , then:

$$\sum_{x \in A} f(x) = \sum_{x \in U} f(x)[x \in A]$$

With $A = \{2, 4\}$, $f(x) = x$, and $U = \{1, 2, 3, 4, 5\}$, both sides equal $2 + 4 = 6$.

However, in statistics and epidemiology, square brackets are already heavily overloaded: they denote [closed intervals](#) $[a, b]$, [conditional expectations](#) $\mathbb{E}[Y \mid X]$ (defined in the probability notes), and matrix delimiters. To prevent visual confusion with expectation brackets or intervals, statistical literature predominantly uses $\mathbb{1}$ or I rather than the bare Iverson bracket.

Summary of indicator notations

Table 1 compares the major notations encountered across the literature.

Table 1: Notations for indicator functions across mathematical and statistical literature

Notation style	Typical syntax	Primary fields	Notes and potential ambiguities
Blackboard bold 1	$\mathbb{1}_A(x), \mathbb{1}(P)$	Modern probability, mathematical statistics	Unambiguous; distinct from matrices and scalars; standard in this book.
Bold numeral 1	$\mathbf{1}_A(x), \mathbf{1}(P)$	Probability theory, measure theory	Can be confused with a vector of ones $\mathbf{1} = (1, \dots, 1)^\top$.
Blackboard bold I	$\mathbb{I}(x \in A), \mathbb{I}(P)$	Econometrics, machine learning, statistics	Clear predicate notation; avoids confusion with numerals.
Letter I	$I_A(x), I(P)$	Classical statistics, epidemiology	Can be confused with the identity matrix I or the Fisher information $\mathcal{I}$, which measures how much a sample tells about a parameter.
Iverson bracket	$[P], [x \in A]$	Computer science, discrete mathematics	Very compact, but square brackets collide with intervals and expectation brackets.

Notation style	Typical syntax	Primary fields	Notes and potential ambiguities
Greek letter χ	$\chi_A(x)$	Real analysis, measure theory	Often termed “characteristic function”; in probability, that name instead means $t \mapsto \mathbb{E}[e^{itX}]$ for a random variable X .

Conventions in this book

In these notes, we standardize on blackboard bold $\mathbb{1}$ via the macros defined in `latex-macros/macros.qmd`:

- `\indic{A}` produces $\mathbb{1}_A$ (set subscript)
- `\indicp{P}` produces $\mathbb{1}(P)$ (predicate in parentheses)
- `\indiccb{P}` produces $\mathbb{1}\{P\}$ (predicate in curly braces)
- `\1{P}` produces 1_P (text numeral with subscript, used in legacy formulas)

Blackboard bold $\mathbb{1}$ is preferred because it avoids all common collisions: it is visually distinct from the scalar 1, the identity matrix I , and the observed and expected Fisher information matrices $(I, \mathcal{I})$.

Key algebraic properties

Indicator functions translate the logical connectives ([definition](#)) “and”, “or”, and “not”, applied to set membership, into ordinary arithmetic on real numbers. One of the identities below uses idempotence:

Definition 0.9 (Idempotent). A number a is **idempotent** [5] if $a \cdot a = a$. A function f with real values is **idempotent** if $f(x) \cdot f(x) = f(x)$ for every x in its [domain](#).

Example 0.7 (Idempotent numbers and indicators).

- 0 and 1 are idempotent: $0 \cdot 0 = 0$ and $1 \cdot 1 = 1$.
- 2 is not idempotent: $2 \cdot 2 = 4 \neq 2$.
- Every indicator function $\mathbb{1}_A$ is idempotent, because each of its values is 0 or 1. For example, with $A = [0, \infty)$,

$$\begin{aligned}\mathbb{1}_A(3.5) \cdot \mathbb{1}_A(3.5) &= 1 \cdot 1 \\ &= 1 \\ &= \mathbb{1}_A(3.5).\end{aligned}$$

The **idempotent matrices** of linear algebra satisfy the same equation with matrix multiplication: $\mathbf{M}^2 = \mathbf{M}$ for a square matrix $\mathbf{M}$.

For subsets A and B of Ω , with **complement** $A^c \stackrel{\text{def}}{=} \Omega \setminus A$, and for every $x \in \Omega$:

- **Intersection (“and”)**: $\mathbb{1}_{A \cap B}(x) = \mathbb{1}_A(x) \cdot \mathbb{1}_B(x)$
- **Union (“or”)**: $\mathbb{1}_{A \cup B}(x) = \mathbb{1}_A(x) + \mathbb{1}_B(x) - \mathbb{1}_A(x) \cdot \mathbb{1}_B(x)$
- **Complement (“not”)**: $\mathbb{1}_{A^c}(x) = 1 - \mathbb{1}_A(x)$
- **Idempotence (Definition 0.9)**: $(\mathbb{1}_A(x))^2 = \mathbb{1}_A(x)$
- **Expectation gives probability**: When Ω is a **sample space** and A is an **event**, the **expectation** of its indicator is the **probability** of the event (all four terms are defined in the Morrison Lab’s probability notes):

$$\begin{aligned}\mathbb{E}[\mathbb{1}_A] &= 0 \cdot \Pr(A^c) + 1 \cdot \Pr(A) \\ &= \Pr(A)\end{aligned}$$

This **identity** turns probabilities into expectations. It is the mathematical foundation for:

- empirical proportions;
- estimators of survival curves, the probability of no event by each time;
- regression models for binary outcomes.

- [1] Wikipedia contributors, “Indicator function — Wikipedia, the free encyclopedia.” 2026. Available: https://en.wikipedia.org/wiki/Indicator_function
- [2] Wikipedia contributors, “Support (measure theory) — Wikipedia, the free encyclopedia.” 2026. Available: [https://en.wikipedia.org/wiki/Support_\(measure_theory\)](https://en.wikipedia.org/wiki/Support_(measure_theory))
- [3] Wikipedia contributors, “Iverson bracket — Wikipedia, the free encyclopedia.” 2026. Available: https://en.wikipedia.org/wiki/Iverson_bracket
- [4] Wikipedia contributors, “Kronecker delta — Wikipedia, the free encyclopedia.” 2026. Available: https://en.wikipedia.org/wiki/Kronecker_delta
- [5] Wikipedia contributors, “Idempotence — Wikipedia, the free encyclopedia.” 2026. Available: <https://en.wikipedia.org/wiki/Idempotence>