Introduction to causal inference

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Contents

1 Introduction 1

2 Individual causal effects 1

1 Introduction

Hernán and Robins (2020) (page vii):

> Unfortunately, the scientific literature is plagued by studies in which the causal question is not explicitly stated and the investigators' unverifiable assumptions are not declared. This casual attitude towards causal inference has led to a great deal of confusion.

Rigorously defining cause and effect is difficult. Fortunately, many humans have strong intuitions about these concepts. We will make cursory attempts at definitions for the basic terms, and leave the finer points to philosophers.

2 Individual causal effects

Definition 2.1 (Action, intervention, exposure, policy, treatment)**.** An **action** (also called an **intervention, exposure, policy,** or **treatment**) is a choice that we consider making.

Definition 2.2 (Random variable)**.** A **random variable** is a variable that may have different values for different individuals and/or for different actions/exposures that an individual might experience. [1]

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[1] adapted from Hernán and Robins (2020) p3

Definition 2.3 (potential outcome, consequence)**.** A **potential outcome** of action a (also called a **consequence** of a) is the value of a random variable Y that would occur if we were to take action a. The potential outcome of action a on random variable Y is written $Y(a)$, Y^a, $Y^{A=a}$ or Y_a in different sources. These notes write it with a subscript, as Y_a.

Definition 2.4 (Factual outcome, observed outcome)**.** A *factual outcome* (or *observed outcome*) is the potential outcome corresponding to an action that was actually taken.

If we consider taking action a or an alternative a', and we actually take action $A = a$, then Y_a is the factual outcome.

Definition 2.5 (Counterfactual outcome)**.** A counterfactual outcome is a potential outcome corresponding to an action that was not actually taken.

If we consider taking action a or an alternative a', and we actually take action $A = a$, then $Y_{a'}$ is a counterfactual outcome.

There might be more than one counterfactual outcome, depending on how many action options were considered, but there can only ever be one factual outcome per random variable.

Definition 2.6 (Cause)**.** Action a **causes** outcome y (or is a cause of y) if:

- outcome y would occur if we were to take action a
  and
- outcome y would not occur if we did not take action a.

In other words, if:

- $Y_a = y$
  and
- $\exists a' \neq a : Y_{a'} \neq y$

Definition 2.7 (effect)**.** The effect of action a on outcome Y, relative to a given alternative action a', is the contrast in potential outcomes, Y_a versus $Y_{a'}$.

Definition 2.8 (Consistency)**.** Consistency is the assumption that if we observe an action a, then the observed outcome Y is equal to the "factual potential outcome" corresponding to action a; in other words, if $A = a$, then $Y_a = Y$.

Definition 2.9 (Exchangeability)**.** Subpopulations defined by exposure X are exchangeable with respect to a potential outcome Y_x if the distribution of Y_x does not depend on the observed exposure X:

$$Y_x \perp\!\!\!\perp X$$

Theorem 2.1. *If subpopulations defined by values of exposure X are exchangeable with respect to potential outcome Y_x, then the expected value of Y_x does not depend on the observed value of X:*

$$\mathbb{E}[Y_x \mid X = x'] = \mathbb{E}[Y_x \mid X = x]$$

Definition 2.10 (Conditional exchangeability)**.** Subpopulations defined by exposure X are exchangeable with respect to a potential outcome Y_x if the distribution of Y_x does not depend on the observed exposure X, conditional on covariate(s) Z:

$$Y_x \perp\!\!\!\perp X \mid \underset{\sim}{Z}$$

Theorem 2.2. *If subpopulations defined by values of exposure X are conditionally exchangeable with respect to potential outcome Y_x given covariate $\underset{\sim}{Z}$, then the expected value of Y_x does not depend on the observed value of X:*

$$\mathbb{E}[Y_x \mid X = x', \underset{\sim}{Z} = \underset{\sim}{z}] = \mathbb{E}[Y_x \mid X = x, \underset{\sim}{Z} = \underset{\sim}{z}]$$

Definition 2.11 (Fundamental problem of causal inference)**.** The *fundamental problem of causal inference* is that only one potential outcome (the factual outcome) can be observed per person (or per sampling unit, more generally) (Holland 1986). The other, counterfactual outcomes, are all missing data, and thus, the causal effects are all missing data as well.

Hernán, Miguel A., and James M. Robins. 2020. *Causal Inference: What If.* Chapman & Hall/CRC. https://miguelhernan.org/whatifbook.

Holland, Paul W. 1986. "Statistics and Causal Inference." *Journal of the American Statistical Association* 81 (396): 945–60. https://doi.org/10.1080/01621459.1986.10478354.